RESEARCH NOTE

Verhulst-Pearl growth model versus Malthusian growth model for in vitro evaluation of lead removal in wastewater by Photobacterium sp [version 1; referees: awaiting peer review]

Lenin Javier Ramirez-Cando, Cesar Ivan Alvarez-Mendoza, Patricia Gutierrez-Salazar
Grupo de Investigación Ambiental para el Desarrollo Sustentable (GIADES), Universidad Politécnica Salesiana, Quito, 170801, Ecuador

Abstract
Mathematical modeling of microbial populations has a long history of application in the fields of ecology and environmental remediation. In the present study, the Verhulst-Pearl growth model and the Malthusian growth model were used to model and understand the kinetics of Photobacterium sp exposed to lead. The results show that goodness of fit of the Verhulst-Pearl growth model was better that the Malthusian growth model. Therefore, the Verhulst-Pearl growth model is considered the best option for proving useful and reliable information about Photobacterium sp kinetics growth in vitro.

Keywords
Photobacterium sp, Lead, Pb, Verhulst-Pearl growth model, Malthusian growth model

Corresponding author: Lenin Javier Ramirez-Cando (biotecnology.ramirez@gmail.com)

Author roles: Ramirez-Cando LJ: Data Curation, Formal Analysis, Project Administration, Writing – Original Draft Preparation; Alvarez-Mendoza CI: Data Curation, Methodology, Resources; Gutierrez-Salazar P: Data Curation, Formal Analysis

Competing interests: No competing interests were disclosed.

Grant information: This work was supported by Universidad Politécnica Salesiana. The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Copyright: © 2018 Ramirez-Cando LJ et al. This is an open access article distributed under the terms of the Creative Commons Attribution Licence, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. Data associated with the article are available under the terms of the Creative Commons Zero "No rights reserved" data waiver (CC0 1.0 Public domain dedication).

How to cite this article: Ramirez-Cando LJ, Alvarez-Mendoza CI and Gutierrez-Salazar P. Verhulst-Pearl growth model versus Malthusian growth model for in vitro evaluation of lead removal in wastewater by Photobacterium sp [version 1; referees: awaiting peer review] F1000Research 2018, 7:491 (https://doi.org/10.12688/f1000research.14420.1)

**Introduction**

It is well-known that some genera of bacteria, such as *Pseudomonas*, *Xanthomonas*, *Ferroxidans*, *Ralstonia*, *Acidobacillus1–3*, have very interesting capacities as heavy metal remediation agents, since they are capable of extracting metals from solid substrates or transform them into harmless forms. Furthermore, bacteria can be used as bio absorbents for the recovery of metals and for the treatment of industrial effluents4,5. For this reason, the application of these technologies and research for continuous improvements is necessary.

In a previous study, the application of *Photobacterium sp* to remove lead from wastewater was shown to be possible6. However, understanding the behavior of this strain within the experimental conditions was not studied. Particularly, computational modeling of the growth kinetics that aims to extract information about interactions between bacteria, nutrients and in this case the pollutant (lead; Pb) could be useful. For this reason, the present study intends to analyze growth of *Photobacterium sp* using two well-known mathematical models, the Verhulst-Pearl growth model and the Malthusian growth model7. These models have a long history of application in the fields of ecology, environmental remediation and industrial fermentation8.

**Methods**

The study uses kinetics data collected from the previous study in Quito-Ecuador, which studied the application of *Photobacterium sp* to remove lead from wastewater6. Briefly in the previous experiments, *Photobacterium sp* was exposed to two concentrations of Pb (20–100 ppm) in a general growth broth. The experiments were performed in times ranging between 0 and 86 hours, under a controlled temperature (25°C) and unsupervised at room temperature, ranging 5–23°C6 at Quito-Ecuador.

**Malthusian growth model**

This model is often referred to as the mathematical exponential law (MEL). It is extensively adopted in the fields of agronomy, ecology or microbiology. The MEL is therefore seldom referred to as the Malthusian Law, which is a widely accepted view to study Malthusian growth in ecology and microbiology.

The following equation was applied to the kinetics data in the present study:

\[ P(t) = P_0 e^{kt} \]

Where, \( P(t) \) is the population in a time, \( P_0 \) is the initial population, \( k \) is the specific growth rate and \( t \) is the time in hours.

**Verhulst-Pearl growth model**

For values of \( t \) in the domain of real numbers, the S-shaped curve is denoted by the model below. The initial stage of growth is approximately stationary and subsequent is nearly exponential; then, as saturation begins, the growth slows, and at maturity, growth stops as shown:

\[ P(t) = P_0 + \frac{A}{1 + e^{-k(t-t_0)}} \]

Where, \( P(t) \) is the population in a time, \( P_0 \) is the initial population, \( km \) is the maximum growth rate and \( t \) is the time in hours. \( t_0 \) represents the time to achieve the middle of the growth and \( A \) represents the carrying capacity of the broth tested.

**Data analysis**

All fittings were performed with sigmaplot 10, using minimum least squares, evaluating the goodness of fit using adjusted R-squared.

**Results**

Fitting a bacteria growth model aims to understand its kinetics. In Table 1 it is evident that Verhulst-Pearl model has a good performance since a R-squared > 0.90, which is considered as acceptable to model bacteria kinetics curves5,9. In these fitted curves, it is noticeable that room temperature parameters differed drastically from 25°C in carrying capacity (A) and middle time (to) to achieve middle biomass concentration. Therefore, these parameters suggest that 25°C controlled temperature increases the performance in both Pb concentrations. Room temperature seems to reduce the growth considering that Quito-Ecuador is located at 2800 m.a.s.l. Moreover, this model explains the reduction in maximum growth ratio due to the increase of Pb concentration. This effect is present at both temperature conditions (Figure 1).

**Table 1. Results of modeling data with Verhulst-Pearl growth model.** Temp=temperature, Pb=lead concentration in ppm.

<table>
<thead>
<tr>
<th>Temp</th>
<th>Pb</th>
<th>A</th>
<th>to</th>
<th>km</th>
<th>Po</th>
<th>R-Sq</th>
</tr>
</thead>
<tbody>
<tr>
<td>25°C</td>
<td>20</td>
<td>6.8E+08</td>
<td>33</td>
<td>0.141</td>
<td>6.0E+07</td>
<td>0.987</td>
</tr>
<tr>
<td>25°C</td>
<td>100</td>
<td>7.0E+08</td>
<td>41</td>
<td>0.081</td>
<td>8.8E+06</td>
<td>0.975</td>
</tr>
<tr>
<td>Room</td>
<td>20</td>
<td>5.2E+08</td>
<td>64</td>
<td>0.113</td>
<td>1.1E+08</td>
<td>0.916</td>
</tr>
<tr>
<td>Room</td>
<td>100</td>
<td>5.5E+08</td>
<td>62</td>
<td>0.085</td>
<td>1.4E+08</td>
<td>0.909</td>
</tr>
</tbody>
</table>
Table 2 shows that the Malthusian growth model had an unacceptable goodness of fit, R-squared < 0.90 for all fitted curves. Therefore, this is considered as an unacceptable model for the bacteria kinetics curves tested. In these fitted curves, it is difficult to see differences in temperature and Pb concentration. However, this model provides information about variations in growth ratio (Figure 2). It would be difficult to extend analysis since this model does not fit as well as the Verhulst-Pearl growth model.

Table 2. Results of modeling data with Malthusian growth model.

<table>
<thead>
<tr>
<th>Temp</th>
<th>Lead (ppm)</th>
<th>Po</th>
<th>km</th>
<th>R-Sq</th>
</tr>
</thead>
<tbody>
<tr>
<td>25°C</td>
<td>20</td>
<td>2.33E+08</td>
<td>0.016</td>
<td>0.718</td>
</tr>
<tr>
<td>25°C</td>
<td>100</td>
<td>1.27E+08</td>
<td>0.022</td>
<td>0.866</td>
</tr>
<tr>
<td>Room</td>
<td>20</td>
<td>7.22E+07</td>
<td>0.025</td>
<td>0.886</td>
</tr>
<tr>
<td>Room</td>
<td>100</td>
<td>1.05E+08</td>
<td>0.021</td>
<td>0.892</td>
</tr>
</tbody>
</table>

Figure 1. Modeling data with Verhulst-Pearl growth model. (A) 25°C, 20ppm lead; (B) Room temperature, 20ppm lead; (C) 25°C, 100ppm lead; and (D) Room temperature, 100ppm lead.
Conclusions

Analysing the goodness of fit reveals that the Verhulst-Pearl growth model is the best option to model the kinetics of *Photobacterium* sp instead of the Malthusian growth model, at least in this particular case. *Photobacterium* sp is also suitable to remove lead from water as shown in Ramirez-Cando *et al.*

The Malthusian model has no manner to estimate Carrying capacity since parameters in the model are developed to explain only the exponential growth phase. Moreover, parameters determined by Verhulst-Pearl growth model are very important in design further research and scaling to preindustrial process in microbiology as well as Monod's model.

Data availability

Dataset 1: Kinetics data obtained in Ramirez-Cando *et al.*

Uploaded with permission of all the authors.

Competing interests

No competing interests were disclosed.

Grant information

This work was supported by Universidad Politecnica Salesiana.

The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

References


4. Sánchez J, Rodríguez J: Fundamentos y Aspectos Microbiológicos;
5. Shaw DR, Dussan J: Mathematical Modelling of Toxic Metal Uptake and Efflux Pump in Metal-Resistant Bacterium Bacillus cereus Isolated From Heavy Crude Oil. Water Air Soil Poll. 2015; 226(4): 112. ISSN 0049-6979. Publisher Full Text


The benefits of publishing with F1000Research:

- Your article is published within days, with no editorial bias
- You can publish traditional articles, null/negative results, case reports, data notes and more
- The peer review process is transparent and collaborative
- Your article is indexed in PubMed after passing peer review
- Dedicated customer support at every stage

For pre-submission enquiries, contact research@f1000.com