Determining the $l_x$ and estimating the force of mortality for children under five in Bangladesh [version 1; peer review: awaiting peer review]

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Abstract

Background: Under-five mortality is a health indicator in population health and the health sciences. Force of death is a more accurate measure of mortality than others, which indicates the situation of mortality when time tends to zero. The purpose of this research is to construct a simulation for $l_x$ (the number of people living at exact age x) for under-five mortality in Bangladesh as a whole, and separately for rural and urban areas, and then estimating the mortality rate in regard to these matched models.

Methods: Secondary data were collected from Bangladesh Demographic and Health Survey 2007. A polynomial model was selected to match the $l_x$ values. To prove the accuracy of the model, the rule of cross-validity was applied.

Results: It has been shown that fourth degree polynomial models can be adjusted to $l_x$ values with explanation of more than 94% variation. It was noted that the mortality rate shows a rapidly decreasing pattern for people aged 0-20 months, a monotonically increasing pattern for those aged 20-53.5 months and then it begins to decrease. It is found that the mortality rate in rural areas is higher than in urban areas of all ages.

Conclusions: It has been shown that the mortality rate in rural areas is higher than for urban areas of all ages, except for those aged 53.5 months. The health situation should therefore be improved to reduce mortality in rural Bangladesh.

Keywords

Children under five years, Force of mortality, Urban and rural mortality, F-test, Bangladesh
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Abbreviations
ANOVA, Analysis of variance; BDHS, Bangladesh Demographic and Health Survey; BBS, Bangladesh Bureau of Statistics; CVPP, cross validity prediction power; IMR, infant mortality rates; EA, enumeration area; MDG 4: Millennium Development Goal 4; NGO, non-governmental organization

Introduction
The mortality rate of infants is a reflection of the well-being and nutritional status of children, and also indicates the socio-economic progress in a given country or region. In truth, it reflects the well-being of mothers, the availability and standard of care for well-being, and how easy it is to receive support in the community and/or country. Child mortality rates and, in particular, infant mortality rates (IMRs) dropped sharply in developed countries, but over the past five decades it has been difficult to lower rates in developing countries such as Bangladesh. It has been found that 29,000 children under the age of five die every day in Bangladesh1.

Five years of previous mortality studies of children under five (2010–2014) is 46 deaths per 1000 live births. Bangladesh achieved its Millennium Development Goal 4 (MDG 4) target for deaths under the age of five, decreasing the rate of child mortality to 48 per 1000 births by 2015. The current infant mortality rate is 38 deaths per 1000 live births, and the mortality rate for children is 8 per 1000 children. In childhood, the risk of death in the first month of life (28 deaths per 1000 live births) is almost three times higher than in the next 11 months (10 deaths per 1000 live births). It has also been found that mortality in the neonatal stage constitutes around 61% of all deaths under the age of five2.

It was noted that in 2007, around 9.2 million live births resulted in death of the child before their fifth birthday3, but in the next twelve months this number fell to 8.8 million4, with 41% of these deaths occurring in newborns5. The IMRs are 44.5 and 76 per 1,000 live births universal for more developed and less developed countries, respectively, indicating that infant death is still high in some vulnerable groups and areas, despite declining trends6. The highest under-five death rates are still observed in sub-Saharan Africa, while South Asia has the second highest rates in the world7. Many countries have not reached MDG 4, but Bangladesh is currently one of the few countries in the world, particularly in South Asia and sub-Saharan Africa, to have reached this goal. Bangladesh decreased death rates from 151 deaths per 1,000 live births in 1990 to 50 deaths per 1,000 live births in 20158. Current studies have revealed that the mortality risk index is 45 per 1,000 live births and that under-five mortality is between 719 and 6510 per 1,000 live births. Although the recent downward trend in infant mortality is notable11, it remains high in Bangladesh due to the high incidence of malnutrition and childhood diseases. One child dies every three or four minutes, resulting in 14 child deaths every hour in Bangladesh12. Deaths usually occur during the neonatal period13.

For this reason, a number of studies have been carried out on the mortality of infants and children and under five; however, in Bangladesh, there have been few studies investigating the mortality rate of infants and children under the age of five. The polynomial model can be used to approximate a complex nonlinear relationship, which is related to the polynomial degree, which depends on the data pattern. In addition, the response variable is estimated at any time within the domain of the matched model in terms of the explanatory variable, i.e. the respondent’s age. Furthermore, mortality is important because it is considered a mortal situation when time tends to zero, but age-specific mortality is the mortality rate within a specific age group. Therefore, the main purpose of this article is to match some models to $l_i$ values for children under five in Bangladesh as a whole, as well as in rural and urban areas separately.

Methods

Data source
To achieve the above-mentioned goals of this study, secondary data on the $l_i$ value for age (in months), i.e. the group data for children under five in Bangladesh as a whole and separately for rural and urban areas, are from Bangladesh Demographic and Health Survey (BDHS) 200714. The 2007 BDHS was representative at national level and included the entire population that did not live in institutional housing units in the country. The survey was used as a sampling frame for the list of enumeration areas (EAs) prepared for the Population and Housing Census 2007, provided by Bangladesh Statistics Office (BBS). The BDHS study was conducted using a two-stage stratified sample of households. A total of 600 areas were initially selected, with 207 clusters in urban areas and 393 in rural areas. A complete list of households was then compiled in all selected areas to provide a sampling framework for the second phase of household selection. In the second phase of the sampling, a systematic sample of an average of 30 households per area was selected to provide statistically reliable estimates of the most important demographic and health variables for the country as a whole, for urban and rural areas individually, and for each of the seven divisions (Dhaka, Rajshahi, Rangpur, Chittagong, Khulna, Barisal and Sylhet). The total sample size is 8,721, of which 5,840 come from rural areas and 2,881 from urban areas during the ten years preceding the survey, in which the sample consists of eligible women, i.e. women who have ever been married and who are aged between 10 and 49 with at least one child. Their data are used as the raw data for this study.

Smoothing of age data
To better adjust the model, the age (in months) associated with the $l_i$ values in Bangladesh and rural and urban areas was smoothed separately using the 4253H smoothing method twice15. The smoothing method was done using Minitab 12.1. It should be noted that the smoothing method is performed to remove age as well as reporting errors before model fitting for better performance as well as better prediction. There are various methods of smoothing, such as the free-hand curve method, the moving average method and 4253H, the double method. The free-hand curve method does not have a mathematical background and the moving average method loses some observations. For this reason, the 4253H method is used, a double method for better efficacy that does not lose any observations.
Model fitting
The $l_i$ values for under-five mortality showed a non-linear distribution. Therefore, an $n$ degree polynomial model is identified in this case and the model is given as:

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u$$  \hspace{1cm} (1)

in which $x$ represents the age in months that is used as explanatory variable in the model, $y$ represents the $l_i$ value, $a_0$ is a constant, $a_i$ is the coefficient of the $x^i$ ($i = 1, 2, 3, ..., n$) and $u$ is the error term of the model\(^{10}\). Here, a suitable $n$ is chosen for which the sum of the squares of the error is the lowest. These models were built using SPSS 22.

Model validation procedure
Cross-validation is a technique used in model selection to better estimate the test error of a predictive model. The idea behind cross-validation is to create a better assessment of a model’s predictive performance. For this, to check the legitimacy of these models over the population, the cross validity prediction power (CVPP), $\rho_{cv}^2$, is employed at this juncture. The mathematical formula for the CVPP is specified below:

$$\rho_{cv}^2 = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k)(n-k-2)} (1 - R^2)$$  \hspace{1cm} (2)

where $n$ is the quantity of classes, $k$ is the number of explanatory variables in the fitted model and $R$ is the correlation between the observed and the fitted values of the response variable. The shrinkage coefficient of the fitted model is the absolute value of $\rho_{cv}^2 - R^2$; where $\rho_{cv}^2$ is the CVPP and $R^2$ is the proportion of variation of this model. Furthermore, the stability situation of $R^2$ of this model is (1 – shrinkage). The estimated CVPP corresponding to $R^2$ and the results for the model fittings are summarized in Table 1. It has been shown that the CVPP can also be used as a model validation technique\(^{17-20}\).

The F-test
To identify the overall significance level of the formulated model and the significance of $R^2$ of the model, the F-test is used in this paper. The formula for the F-test is given as:

$$F = \frac{(p-1)}{(1-R^2)} \frac{(n-p)}{(n-p)}$$

where $p$ is the number of parameters of the fitted model, $n$ is the number of cases and $R^2$ is the coefficient of determination of the fitted model\(^{17}\).

Instantaneous force of mortality ($\mu_x$)
The instantaneous force of mortality at age $x$ is defined as the ratio of the instantaneous rate of decrease in $l_i$ to the value of $l_i$. It is denoted by $\mu_x$ and, in the limiting case, it is given using differential calculus by:

$$\mu_x = \lim_{t \to 0} \frac{\frac{d}{dt} l_i}{t l_i} = -\frac{1}{l_i} \frac{d}{dx} (l_i) = -\frac{d}{dx} (\ln l_i) \hspace{1cm} \text{[28, 29]}$$

In brief, it is also called the force of mortality. In fact, the force of mortality is a more accurate measure in mortality studies than age-specific death rates. It measures the mortality situation when time tends to zero. This is an instantaneous measurement, rather than an interval measurement.

Table 1. The results of CVPP and information on model fittings.

<table>
<thead>
<tr>
<th>Models</th>
<th>$n$</th>
<th>$K$</th>
<th>Parameters</th>
<th>p-values</th>
<th>$R^2$</th>
<th>$\rho_{cv}^2$</th>
<th>Shrinkage</th>
<th>F-test (calculated and tabulated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model for Bangladesh</td>
<td>11</td>
<td>5</td>
<td>$a_0$</td>
<td>$&lt;0.0001$</td>
<td>0.94553</td>
<td>0.821731</td>
<td>0.123798</td>
<td>26.08 and 9.15 with (4,6) df</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_1$</td>
<td>0.001102</td>
<td></td>
<td></td>
<td></td>
<td>at 1% level of significance.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_2$</td>
<td>0.007429</td>
<td></td>
<td></td>
<td></td>
<td>Significant.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_3$</td>
<td>0.020427</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_4$</td>
<td>0.038182</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model for rural areas</td>
<td>11</td>
<td>5</td>
<td>$a_0$</td>
<td>$&lt;0.0001$</td>
<td>0.94754</td>
<td>0.828309</td>
<td>0.11923</td>
<td>27.093 and 9.15 with (4,6) df</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_1$</td>
<td>0.001261</td>
<td></td>
<td></td>
<td></td>
<td>at 1% level of significance.</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$a_2$</td>
<td>0.008829</td>
<td></td>
<td></td>
<td></td>
<td>Significant.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_3$</td>
<td>0.023974</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_4$</td>
<td>0.044073</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model for urban areas</td>
<td>11</td>
<td>5</td>
<td>$a_0$</td>
<td>$&lt;0.0001$</td>
<td>0.94241</td>
<td>0.811537</td>
<td>0.13088</td>
<td>24.548 and 9.15 with (4,6) df</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_1$</td>
<td>0.000799</td>
<td></td>
<td></td>
<td></td>
<td>at 1% level of significance.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_2$</td>
<td>0.005044</td>
<td></td>
<td></td>
<td></td>
<td>Significant.</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>$a_3$</td>
<td>0.014332</td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$a_4$</td>
<td>0.027845</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

CVPP: cross validity prediction power; df, degrees of freedom.
Results

The polynomial model is used to fit the age associated with the $l_x$ values for children under five in Bangladesh in rural and urban areas, and these fitted models are presented (Figure 1, Figure 2, Figure 3) below:

i) For Bangladesh, $y=950.1666-39.3148x+2.1053x^2-0.0449x^3+0.0003x^4$; t-value: (42.33616) (-5.84934) (3.96269) (-3.12604) (2.64699)

ii) For rural areas, $y=966.3290-26.1108x+1.3809x^2-0.0294x^3+0.0002x^4$; t-value: (63.16859) (-5.69949) (3.81335) (-3.00112) (2.54014)

iii) For urban areas, $y=984.0909-13.2389x+0.7251x^2-0.0155x^3+0.0001x^4$; t-value: (138.4389) (-6.2189) (4.3088) (-3.4093) (2.8858)

The results of the fitted models and the CVPP corresponding to the $R^2$ of these fitted models of $l_x$ values for children under five in Bangladesh and in the rural and urban areas are presented in Table 2. It seems that the constructed models (i) to (iii) are cross-validated, and their shrinkages are also shown in Table 2. These imply that the fitted models (i) to (iii) are more than 81% stable and this is demonstrated in Table 2.

In addition, it can be seen that the parameters of these models are significant, with more than 94% of the explained variance, which is also shown in Table 3. The stability position for $R^2$ of these matched models is over 87%. The calculated values for the F-test, i.e. the analysis of variance (ANOVA) for these matched models with (5, 7) df, are 26.08, 27.09 and 24.548 and are displayed in Table 2, while the corresponding tabular values are 9.15 for models (i) to (iii) at a significance level of 1%. Consequently, on the basis of these test statistics, it is concluded that the F-test is highly significant, and therefore, that these models are highly significant. That is why the fitting of the models is good. Hence, forecasts based on these models are shown in Table 1.

It should be mentioned here that other more typical models, such as the linear model ($R^2 = 0.53$), exponential ($R^2 = 0.55$), square ($R^2 = 0.73$) and the cubic model ($R^2 = 0.85$), were also used for the Bangladesh data set, but they did not show a better fit due to the low coefficient of determination.

The ages associated with force of mortality for children aged under five in Bangladesh, and in rural and urban areas, are estimated from the fitted model of the $l_x$ values that are presented in Table 3 and shown in Figure 4 to Figure 6.

In Table 3, as well as in the numbers described above, it can be seen that the mortality of children under five in Bangladesh, as well as in rural and urban areas, shows a rapidly decreasing pattern at 0–20 months, followed by a monotonically increasing pattern for age from 20 to 53.5 months and then it begins to decrease. In addition, it has been demonstrated that the mortality rate in rural areas is higher than for urban areas of all ages excluding those aged 53.5 months. All three cases show a similar pattern.

Discussion

Explanation of the empirical age patterns of death using these models is one of the most important topics in population studies, especially in demographics. In this study, the fourth degree polynomial models are well-suited to $l_x$ values for children ages under five, with an explanation for the huge proportion of variation. The mortality of infants and children in Bangladesh is significantly reduced, but the future
increase in life expectancy requires an additional reduction in mortality. This study estimates how age is related to the mortality of children under the age of five in Bangladesh as a whole and in rural and urban areas separately. We expect these recent mortality findings to convince the government, non-governmental organizations and those responsible for policy planning of the need for a planned increase in socio-demographic development and health care facilities. For this reason, the survival function taken from the data is selected as the raw material in this study. The mortality force is the ratio between the instantaneous rate of decrease in $l_x$ and the value of $l_x$, which is obtained from the functional dependence between $l_x$ and age $x$. 

**Figure 2.** Smoothed and predicted $l_x$ values for children under five in rural areas in Bangladesh. X axis represents age group and Y axis represents $l_x$.

**Figure 3.** Smoothed and predicted $l_x$ values for children under five in urban areas in Bangladesh. X axis represents age group and Y axis represents $l_x$. 
Table 2. The observed, smoothed and predicted survival function ($l_x$ values) for children under five in Bangladesh.

<table>
<thead>
<tr>
<th>Age in Months</th>
<th>Bangladesh</th>
<th>Rural</th>
<th>Urban</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Smoothed</td>
<td>Predicted</td>
</tr>
<tr>
<td>0</td>
<td>1000</td>
<td>992</td>
<td>950</td>
</tr>
<tr>
<td>1.5</td>
<td>835</td>
<td>871</td>
<td>896</td>
</tr>
<tr>
<td>4</td>
<td>787</td>
<td>791</td>
<td>824</td>
</tr>
<tr>
<td>7</td>
<td>751</td>
<td>752</td>
<td>764</td>
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<tr>
<td>10</td>
<td>733</td>
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<td>726</td>
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<tr>
<td>14.5</td>
<td>725</td>
<td>723</td>
<td>700</td>
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<tr>
<td>20.5</td>
<td>714</td>
<td>713</td>
<td>700</td>
</tr>
<tr>
<td>31.5</td>
<td>702</td>
<td>701</td>
<td>717</td>
</tr>
<tr>
<td>41.5</td>
<td>686</td>
<td>688</td>
<td>699</td>
</tr>
<tr>
<td>53.5</td>
<td>677</td>
<td>678</td>
<td>657</td>
</tr>
<tr>
<td>60</td>
<td>669</td>
<td>669</td>
<td>679</td>
</tr>
</tbody>
</table>

Table 3. Instantaneous force of mortality for children under five in Bangladesh, in rural and in urban areas.

<table>
<thead>
<tr>
<th>Age in Months</th>
<th>Bangladesh</th>
<th>Rural</th>
<th>Urban</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.041377</td>
<td>0.027021</td>
<td>0.013453</td>
</tr>
<tr>
<td>1.5</td>
<td>0.037172</td>
<td>0.023828</td>
<td>0.011562</td>
</tr>
<tr>
<td>4</td>
<td>0.029802</td>
<td>0.018618</td>
<td>0.008661</td>
</tr>
<tr>
<td>7</td>
<td>0.020994</td>
<td>0.012862</td>
<td>0.005672</td>
</tr>
</tbody>
</table>

Figure 4. Instantaneous force of mortality for children under five in Bangladesh. X axis represents age in months and Y axis represents force of mortality.
Figure 5. Instantaneous force of mortality for children under five in rural areas in Bangladesh. X axis represents age in months and Y axis represents force of mortality.

Figure 6. Instantaneous force of mortality for children under five in urban areas in Bangladesh. X axis represents age in months and Y axis represents force of mortality.
Mortality rate should be treated as the probability of death at a given moment, taking into account survival until that time. It was reported that the $I_x$ values for the Bangladeshi male population are in line with the four-parameter cubic polynomial model, and it has been observed that the $I_x$ values for the Bangladeshi population are consistent with the third degree polynomial model. However, in this study, efforts were made to focus on which mathematical model best suits the relationship between age and $I_x$ values for minors in Bangladesh as a whole, and in rural and urban areas separately. For this purpose, the polynomial model is chosen. It should be mentioned that a number of studies were carried out using the polynomial model. One of the limitations of the survey is that, except for a few questions, almost all information collected in BDHS surveys is subject to reporting errors.

Conclusions
In this study, it was found that the relationship between age and $I_x$ values for children under five in Bangladesh results from a four-level polynomial model containing five parameters. The mortality rate shows a rapidly decreasing pattern at 0–20 months and a monotonically increasing pattern between 20 and 53.5 months, and then begins to decrease in all cases. This research result has fundamental strategic implications, especially for educational and awareness program requirements for a real decrease in child mortality and compliance with public health interventions. The researcher recommends additional research, taking into account these unobserved issues that are probably associated with the death of infants and children, to better understand the relationship between family and public factors and the death of a child in Bangladesh. Intervention programs and policies should focus on this research to improve the health consequences for children, in order to achieve future improvement of the situation in Bangladesh as well as in other areas such as sub-Saharan Africa.

Ethical approval
Ethical approval for this study was not applicable, since ethical approval for the collection of data was previously approved for BDHS.

Data availability
Underlying data
The data from BDHS 2007 are free to access (https://dhsprogram.com/data/dataset/Bangladesh_Standard-DHS_2007.cfm?flag=0); however, before you can download data, users must register as a DHS data user for reasons laid out on the DHS website (https://www.dhsprogram.com/data/Registration-Rationale.cfm). Dataset access is only granted for legitimate research purposes (https://dhsprogram.com/data/new-user-registration.cfm).

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References

19. Islam MR: Modelling age structure and ASDRs for human population of both

Page 9 of 11


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