RESEARCH ARTICLE

Modified Kibria-Lukman (MKL) estimator for the Poisson Regression Model: application and simulation [version 1; peer review: 3 approved with reservations]

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First published: 08 Jul 2021, 10:548
https://doi.org/10.12688/f1000research.53987.1

Abstract

Background: Multicollinearity greatly affects the Maximum Likelihood Estimator (MLE) efficiency in both the linear regression model and the generalized linear model. Alternative estimators to the MLE include the ridge estimator, the Liu estimator and the Kibria-Lukman (KL) estimator, though literature shows that the KL estimator is preferred. Therefore, this study sought to modify the KL estimator to mitigate the Poisson Regression Model with multicollinearity.

Methods: A simulation study and a real-life study were carried out and the performance of the new estimator was compared with some of the existing estimators.

Results: The simulation result showed the new estimator performed more efficiently than the MLE, Poisson Ridge Regression Estimator (PRE), Poisson Liu Estimator (PLE) and the Poisson KL (PKL) estimators. The real-life application also agreed with the simulation result.

Conclusions: In general, the new estimator performed more efficiently than the MLE, PRE, PLE and the PKL when multicollinearity was present.

Keywords

Linear regression model, generalized regression model, Ridge estimator, Liu estimator, KL estimator.

Open Peer Review

Reviewer Status

Invited Reviewers

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1. Mohammad Arashi, Ferdowsi University of Mashhad, Mashhad, Iran
2. Muhammad Amin, University of Sargodha, Sargodha, Pakistan
3. Nimet Özbay, Çukurova University, Adana, Turkey

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Introduction
A special case of the Generalized Linear Models (GLM) is the Poisson Regression Model (PRM) which is generally applied for count or frequency data modelling. It is employed to model the relationship between a response variable and one or more independent variable where the response variable denotes a rare event or count data. The response variable also takes the form of a non-negative variable, and it is applicable in the following fields: economics, health, social and physical sciences. The Maximum Likelihood Estimation (MLE) method is popularly used to estimate the regression coefficient in a PRM. In both a Linear Regression Model (LRM) and Generalized Linear Model (GLM), MLE suffers a setback when the independent variables are correlated, which implies multicollinearity. Multicollinearity effects include large variance and regression coefficient covariances, negligible t-ratio and a high coefficient of determination (R-square) values. Alternative estimators to the MLE in the linear regression model include the ridge regression estimator by Hoerl and Kennard (1970), Liu estimator by Liu (1993), Liu-type estimator by Liu (2003), two-parameter estimator by Özkale and Kaciranlar (2007), r-d class estimator Kaçiranlar and Sakallioğlu (2007), k-d class estimator Sakallioğlu and Kaciranlar (2008), a two-parameter estimator by Yang and Chang (2010), modified two-parameter estimator by Dorugade (2014), modified ridge-type estimator by Lukman et al. (2019), modified Liu estimator by Lukman et al. (2020), Kibria-Lukman (KL) estimator by Kibria and Lukman (2020), modified new two-parameter estimator by Ahmad and Aslam (2020), the modified Liu ridge type estimator by Aslam and Ahmad (2020) and the DK estimator by Dawoud and Kibria (2020) among others. Researchers have extended some of these existing estimators in LRM to the PRM. Mansson et al. (2012) introduced the Liu estimator into the PRM. The modified jackknifed ridge estimator for the PRM was introduced by Türkan and Özel (2016). The ridge estimator was introduced into the PRM by Mansson and Shukur (2011). A new two-parameter for PRM was developed by Asar and Genç (2017). Recently, Poisson KL estimator was developed by Lukman et al. (2021) for combating multicollinearity in the PRM.

In this study, we modified the KL estimator to handle multicollinearity in PRM. Furthermore, we compared the performance of the estimator with the Poisson Maximum Likelihood Estimator (PMLE), Poisson Ridge Regression Estimator (PRE), Poisson Liu Estimator (PLE) and the Poisson KL estimator (PKLE).

Methods
Given that the response variable, $y_i$ is in the form of count data, then it is assumed to follow a Poisson distribution as

$$P_i = \exp(x_i \beta)$$

such that $x_i$ is the $i^{th}$ row of $X$ which is a $n \times (p+1)$ data matrix with $p$ independent variables and $\beta$ is a $(p+1) \times 1$ vector of coefficients. The log likelihood of the model is given as:

$$l(\mu; y) = \sum_{i=1}^{n} \exp(x_i \beta) + \sum_{i=1}^{n} y_i \log(\exp(x_i \beta)) + \log \left( \prod_{i=1}^{n} y_i ! \right)$$

(2.1)

The most common method of maximizing the likelihood function is to use the iterated weighted least squares (IWLS) algorithm which results to:

$$\hat{\beta}_{MLE} = \left( X' \tilde{V} X \right)^{-1} \left( X' \tilde{V} z \right)$$

(2.2)

where $\tilde{V} = \text{diag}[\hat{\mu}_i]$ and $\tilde{z}$ is a vector while the $i^{th}$ element equals $\tilde{z}_i = \log(\hat{\mu}_i) + \frac{y_i - \hat{\mu}_i}{\hat{\mu}_i}$.

The MLE is normally distributed with a covariance matrix that is equivalent to the inverse of the second derivative as:

$$\text{Cov}(\hat{\beta}_{MLE}) = \left( -E \left( \frac{\partial^2 l}{\partial \beta \partial \beta} \right) \right)^{-1} = \left( X' \tilde{V} X \right)^{-1}$$

(2.3)

and the mean square error is given as:

$$E(\hat{\beta}_{MLE}) = E(\hat{\beta}_{MLE} - \beta)'(\hat{\beta}_{MLE} - \beta) = \text{tr}(X' \tilde{V} X)^{-1} = \sum_{j=1}^{p} \frac{1}{\lambda_j}$$

where $\lambda_j$ is the j$^{th}$ eigen value of the $X' \tilde{V} X$ matrix.

The Poisson Ridge Estimator (PRE) was introduced by Mansson and Shukur (2011) as a solution to multicollinearity in PRM. The estimator is defined as follows:

$$\hat{\beta}_{PRE} = \left( X' \tilde{V} X + kI \right)^{-1} X' \tilde{V} X \hat{\beta}_{MLE}$$

(2.4)
where \( k = \max \frac{1}{m} \) and \( \sqrt{\pi^2} \).

The mean square error (MSE) is:

\[
MSE(\hat{\beta}_{PRE}) = \sum_{j=1}^{p} \frac{\lambda_j}{(\lambda_j + k)^2} + k^2 \sum_{j=1}^{p} \frac{\hat{\sigma}_j^2}{(\lambda_j + k)^2}
\]

\[\text{(2.5)}\]

Mansson et al. (2012) developed the Liu estimator to the Poisson regression model as:

\[
\hat{\beta}_{PLE} = \left( X'\hat{\Sigma}X + I \right)^{-1} (X'\hat{\Sigma}X + d\hat{\alpha}) \hat{\beta}_{ML}
\]

where

\[
\hat{d} = \max \left( 0, \frac{\hat{\sigma}_j^2 - 1}{\hat{\alpha}_j^2 + \frac{1}{\lambda_j}} \right)
\]

\[\text{(2.7)}\]

The means square error for the Liu estimator is defined as:

\[
MSE(\hat{\beta}_{PLE}) = \sum_{j=1}^{p} \frac{(\lambda_j + d)^2}{\lambda_j(\lambda_j + 1)^2} + (d - 1)^2 \sum_{j=1}^{p} \frac{\lambda_j}{(\lambda_j + 1)^2}
\]

\[\text{(2.8)}\]

where \( \lambda_j \) is the \( j \)th eigenvalue of \( X'\hat{\Sigma}X \) and \( \alpha_j \) is the \( j \)th element of \( \alpha \).

The KL estimator was proposed by Kibria and Lukman (2020) as a means of mitigating the effect of multicollinearity on parameter estimation. The estimator is defined as

\[
\hat{\beta}_{KL} = \left( X'X + kI \right)^{-1} (X'X - k) \hat{\beta}_{MLE}
\]

By means of extension, the Poisson K-L estimator was proposed by Lukman et al. (2021) as follows:

\[
\beta_{PKL} = \left( X'\hat{\Sigma}X + k \right)^{-1} (X'\hat{\Sigma}X - k) \hat{\beta}_{MLE}
\]

\[\text{(2.10)}\]

\[
MSE(\hat{\beta}_{PKL}) = \sum_{j=1}^{p} \left( \frac{(\lambda_j - k)^2}{\lambda_j(\lambda_j + k)^2} \right) + 4k^2 \sum_{j=1}^{p} \frac{\lambda_j^2}{(\lambda_j + k)^2}
\]

\[\text{(2.11)}\]

The Poisson Modified KL estimator (PMKL)

The proposed estimator is obtained as follows: \( \hat{\beta}_{MLE} \) in equation (2.10) is replaced with the ridge estimator. Thus, we have:

\[
\beta_{MKL} = (X'X + k)^{-1} (X'X - k)(X'X + k)^{-1}X'y
\]

\[\text{(2.12)}\]

The properties of the new estimator include:

\[
E(\hat{\beta}_{MKL}) = (X'X + kI)^{-1} (X'X - kI)(X'X + kI)^{-1}X'X\beta
\]

\[\text{(2.13)}\]

\[
Bias(\hat{\beta}_{MKL}) = (X'X + kI)^{-1} (X'X - kI)(X'X + kI)^{-1}X'X\beta - \beta
\]

\[
= (X'X + kI)^{-2} k[-3X'X - kI]\beta
\]

\[\text{(2.14)}\]
The bias can be written in scalar form as:

$$\text{Bias}(\hat{\beta}_{\text{MKL}}) = k \sum_{j=1}^{p} \frac{(-3\lambda_j - k)\beta}{(\lambda_j + k)^2}$$  \hspace{1cm} (2.15)

$$V(\hat{\beta}_{\text{MKL}}) = \sigma^2 \left( X'X + kI \right)^{-1} (X'X - kI) V(\hat{\beta}_{\text{MKL}})$$  \hspace{1cm} (2.16)

Thus, the MSE is obtained as:

$$MSE(\hat{\beta}_{\text{MKL}}) = \sigma^2 \left[ 2 \sum_{j=1}^{p} \frac{\lambda_j(\lambda_j - k)^2}{(\lambda_j + k)^2} + k^2 \sum_{j=1}^{p} \frac{(3\lambda_j + k)^2\beta^2}{(\lambda_j + k)^4} \right]$$  \hspace{1cm} (2.18)

The proposed estimator in (2.13) is extended to the PRM. It is referred to as the Poisson modified KL (PMKL) estimator and defined as:

$$\hat{\beta}_{\text{PMKL}} = \left( X'\tilde{V}X + k \right)^{-1} \left( X'\tilde{V}X - k \right) \left( X'\tilde{V}X + k \right)^{-1} X'\tilde{V}X \hat{\beta}_{\text{MLE}}$$  \hspace{1cm} (2.19)

The mean square error of the PMKL is defined as:

$$MSE(\hat{\beta}_{\text{PMKL}}) = \sum_{j=1}^{p} \frac{\lambda_j(\lambda_j - k)^2}{(\lambda_j + k)^2} + k^2 \sum_{j=1}^{p} \frac{(3\lambda_j + k)^2\alpha_j^2}{(\lambda_j + k)^4}$$  \hspace{1cm} (2.20)

The following lemmas are adopted for theoretical comparisons among the estimators.

**Lemma 2.1** Let $A$ be a positive definite (pd) matrix, that is, $A > 0$, and $a$ be some vector, then $A - aa' \geq 0$ if and only if (iff) $a'\alpha^{-1}a \leq 1$ (Farebrother, 1976).

**Lemma 2.2** $MSEM(\hat{\beta}_1) - MSEM(\hat{\beta}_2) = \sigma^2 D + b_1 b_2' - b_2 b_2' > 0$

if and only if $b_2'[\sigma^2 D + b_1 b_1']^{-1} b_2 < 1$ where $MSEM(\hat{\beta}_j) = Cov(\hat{\beta}_j) + b_j b_j$ (Trenker and Toutenburg, 1990).

**Theorem 2.1**: $\hat{\alpha}_{\text{PMKL}}$ is preferred to $\hat{\alpha}_{\text{MLE}}$ iff, $MSEM(\hat{\alpha}_{\text{MLE}}) - MSEM(\hat{\alpha}_{\text{PMKL}}) > 0$ provided $k > 0$.

**Proof**

$$V(\hat{\alpha}_{\text{PMLE}}) - V(\hat{\alpha}_{\text{PMKL}}) = Q\text{diag} \left\{ \frac{1}{\lambda_j} \frac{\lambda_j(\lambda_j - k)^2}{(\lambda_j + k)^2} \right\} Q'$$

It is observed that $(\lambda_j + k)^4 - \lambda_j^2(\lambda_j - k)^2 > 0$ such that the expression above is non-negative for $k > 0$.

**Theorem 2.2**: $\hat{\alpha}_{\text{PMKL}}$ is preferred to $\hat{\alpha}_{\text{PRE}}$ iff, $MSEM(\hat{\alpha}_{\text{PRE}}) - MSEM(\hat{\alpha}_{\text{PMKL}}) > 0$ provided $k > 0$.

**Proof**

$$V(\hat{\alpha}_{\text{PRE}}) - V(\hat{\alpha}_{\text{PMKL}}) = Q\text{diag} \left\{ \frac{1}{\lambda_j} \frac{\lambda_j(\lambda_j - k)^2}{(\lambda_j + k)^2} \right\} Q'$$

We can observe that the difference of the variance of the estimator is non-negative since $(\lambda_j + k)^4 \lambda_j - \lambda_j^2(\lambda_j - k)^2 > 0$ for $k > 0$. 

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Theorem 2.3: $\hat{\alpha}_{PMKL}$ is preferred to $\hat{\alpha}_{PLE}$ iff, $MSEM(\hat{\alpha}_{PLE}) - MSEM(\hat{\alpha}_{PMKL}) > 0$ provided $k > 0$ and $0 < d < 1$.

Proof

$$V(\hat{\alpha}_{PLE}) - Cov(\hat{\alpha}_{PMKL}) = Q\text{diag} \left\{ \frac{(\hat{\alpha}_j + d)^2}{\hat{\alpha}_j(\hat{\alpha}_j + 1)} - \frac{\hat{\lambda}_j(\hat{\lambda}_j - k)^2}{(\hat{\lambda}_j + k)^4} \right\}_{j=1}^p Q'$$

The difference of the variance is non-negative since

$$(\hat{\lambda}_j + k)(\hat{\lambda}_j + d) - \hat{\lambda}_j(\hat{\lambda}_j + 1)(\hat{\lambda}_j - k) > 0$$

for $0 < d < 1$ and $k > 0$.

Theorem 2.4: $\hat{\alpha}_{PMKL}$ is preferred to $\hat{\alpha}_{PCL}$ iff, $MSEM(\hat{\alpha}_{PCL}) - MSEM(\hat{\alpha}_{PMKL}) > 0$ provided $k > 0$.

Proof

$$V(\hat{\alpha}_{PCL}) - Cov(\hat{\alpha}_{PMKL}) = Q\text{diag} \left\{ \frac{(\hat{\lambda}_j - k)^2}{\hat{\lambda}_j(\hat{\lambda}_j + k)} - \frac{\hat{\lambda}_j(\hat{\lambda}_j - k)^2}{(\hat{\lambda}_j + k)^4} \right\}_{j=1}^p Q'$$

The difference of the variance is non-negative since $(\hat{\lambda}_j + k)(\hat{\lambda}_j - k) - \hat{\lambda}_j(\hat{\lambda}_j - k) > 0$ for $k > 0$.

Selection of biasing parameter

The biasing parameter $k$ for the estimator is obtained by differentiating the MSE with respect to $k$ and obtained as:

$$k_{MKL} = \min \left[ \frac{\sqrt{(3\hat{\lambda}_i\alpha^2 + \sigma^2)^2 + 4\alpha^2\sigma^2\lambda_i - 3\hat{\lambda}_i\alpha^2 + \sigma^2}}{2\beta^2} - \frac{(3\hat{\lambda}_i\alpha^2 + \sigma^2)}{2\lambda_i\alpha_j^2 + 1} \right]$$

(2.21)

The shrinkage parameter estimated by Mansson and Shukur, (2011) and Kibria and Lukman (2020) was also adopted for this study as listed:

$$k_1 = \frac{1}{\max(\alpha_j^2)}$$

(2.22)

$$k_2 = \frac{p}{\sum (2\alpha_i^2 + \lambda_j^4)}$$

(2.23)

$$k_3 = \min \left( \frac{\lambda_i}{2\lambda_i\alpha_j^2 + 1} \right)$$

(2.24)

$k_1$ is the biasing parameter for PMKL1, while $k_2$ and $k_3$ are the biasing parameters for PMKL2 and PMKL3.

Simulation Design and Real-Life Application

Simulation study and result

In this section, a simulation study is carried out to compare the performance of the different estimators. The generation of the dependent variables are done using pseudo-random numbers from Po($\mu_i$) where $\mu_i = \alpha^\beta_i$, $i = 1, 2, \ldots, n$ and $X_i$ is the $i$th row of the design matrix with $\beta = (\beta_1, \beta_2, \ldots, \beta_p)$ being the coefficient vector. The generation of the independent variables with different levels of correlation is obtained using

$$x_{ij} = (1 - \rho^2)^{1/2}z_{ij} + \rho z_{ip}$$

(3.1)

where $\rho$ is the level of multicollinearity between the independent variables (Kibria et al. 2015; Kibria and Banik, 2016; Lukman et al., 2019b, Lukman et al. 2020b). $z_{ij}$ are pseudo-random numbers generated using the standard normal distribution such that $i$ ranges from 1 to $n$ and $j$ from 1 to $p$. As a common restriction used in simulation studies, it is assumed that $\sum_{j=1}^{p}\beta_j^2 = 1$ and $\beta_1 = \beta_2 = \ldots = \beta_p$. Also, the effect of the intercept value is also being investigated as values are taken to be 1, 0 and -1 (Kibria et al. 2014). The different levels of correlation taken are 0.8, 0.9, 0.95, 0.99 and
0.999. The other factors varied in the simulation study are the sample size \( n \) and the number of independent variable \( p \). We assume \( n = 50, 100 \) and \( 200 \) observations and \( p = 4 \) and \( 8 \) independent variables.

The simulation results in Tables 1 to 6 that for each of the estimators, the simulated MSE values increase as the multicollinearity level increases, keeping other factors constant. There is also an increase in the mean square error as the sample size increases for all estimators compared while other factors were kept constant. As the intercept values varied from -1 to +1, the values of the mean square error reduced for all estimators. Result shows that the PMKL1 performed best with minimum MSE at varying sample sizes. It was closely followed by PMKL2. They are both considered more suitable for estimation of parameters in the Poisson regression model than the MLE as it performed worst when multicollinearity is a challenge. In general, the PMKL1 estimator consistently performed more efficiently than the MLE, PRE, PLE and the PKL estimators.

**Real Life Application**

Having carried out a simulation study, the efficacy of the proposed estimator needs to be further investigated by considering a real-life application. The Poisson regression model has been applied to the aircraft damage dataset initially by Myers et al. (2012) and subsequently by other researchers such as Asar and Genc (2017) and Amin et al. (2020) among others. By following the Pearson chi-square goodness of fit test, Amin et al. (2020) was able to ascertain that the data fits a Poisson regression model. The test confirms the suitability of the response variable to Poisson distribution with P-value of 6.898122 (0.07521). The dataset provides some detail on two separate aircrafts: The McDonnell Douglas A-4 Skyhawk and the A-6 Grumman Intruder. The dependent variable denotes the number of locations with damage on the aircraft and this follows a Poisson distribution (Asar and Genc, 2017; Amin et al., 2020). The data set has three explanatory variables, \( X_1 \) shows the type of aircraft which makes the outcome binary (A-4 is coded as 0 and A-6 is coded as 1). \( X_2 \) is the bomb load in tons and \( X_3 \) is the number of months of aircrew experience. Meyers et al. (2012) was able to ascertain that the data set is greatly affected by multicollinearity. The eigenvalues of the matrix \( X \) were obtained as 4.3333, 374.8961 and 2085.2251. The condition number of 219.3654 was also obtained which is an indication of the problem of multicollinearity since it is greater than 30 (Asar and Genc, 2017). The performance of the estimators is judged based on the mean square error of each of the estimators.

From Table 7, it is evident that all of the regression coefficients had identical signs. The estimator with the highest mean squared error is the MLE due to the presence of multicollinearity. The suggested estimator (PMKL1, PMKL2, PMKL3) has the lowest MSE that has established its dominance. We also observed that the performance of the estimator is highly dependent on the biasing parameter \( k \).

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MLE = Maximum Likelihood Estimator; PRE = Poisson Ridge Estimator; PLE = Poisson Liu Estimator; PKL = Poisson Kibria-Lukman Estimator; PMKL1 = Poisson Modified Kibria Lukman Estimator 1; PMKL2 = Poisson Modified Kibria Lukman Estimator 2; PMKL3 = Poisson Modified Kibria Lukman Estimator 3.
### Table 2. Simulation result for mean square error (MSE) when $P = 4$ and intercept = 0.

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MLE = Maximum Likelihood Estimator; PRE = Poisson Ridge Estimator; PLE = Poisson Liu Estimator; PKL = Poisson Kibria-Lukman Estimator; PMKL1 = Poisson Modified Kibria Lukman Estimator 1; PMKL2 = Poisson Modified Kibria Lukman Estimator 2; PMKL3 = Poisson Modified Kibria Lukman Estimator 3.

### Table 3. Simulation result for mean square error (MSE) when $P = 4$ and intercept = -1.

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MLE = Maximum Likelihood Estimator; PRE = Poisson Ridge Estimator; PLE = Poisson Liu Estimator; PKL = Poisson Kibria-Lukman Estimator; PMKL1 = Poisson Modified Kibria Lukman Estimator 1; PMKL2 = Poisson Modified Kibria Lukman Estimator 2; PMKL3 = Poisson Modified Kibria Lukman Estimator 3.
### Table 4. Simulation result for mean square error (MSE) when $P = 8$ and intercept = 1.

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MLE = Maximum Likelihood Estimator; PRE = Poisson Ridge Estimator; PLE = Poisson Liu Estimator; PKL = Poisson Kibria-Lukman Estimator; PMKL1 = Poisson Modified Kibria Lukman Estimator 1; PMKL2 = Poisson Modified Kibria Lukman Estimator 2; PMKL3 = Poisson Modified Kibria Lukman Estimator 3.

### Table 5. Simulation result for mean square error (MSE) when $P = 8$ and intercept = 0.

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</table>

MLE = Maximum Likelihood Estimator; PRE = Poisson Ridge Estimator; PLE = Poisson Liu Estimator; PKL = Poisson Kibria-Lukman Estimator; PMKL1 = Poisson Modified Kibria Lukman Estimator 1; PMKL2 = Poisson Modified Kibria Lukman Estimator 2; PMKL3 = Poisson Modified Kibria Lukman Estimator 3.
The parameters in the PRM are commonly estimated using the Maximum Likelihood Estimator. However, literature has shown that the estimator suffers a setback when the explanatory variables are correlated. This problem led to the implementation of alternative estimators with single shrinkage parameters such as the Poisson Ridge Regression Estimator (PRE), Poisson Liu Estimator (PLE) and the Poisson KL Estimator (PKL). The KL estimator was generally preferred to the ridge regression and Liu estimator in the linear regression model. According to Lukman et al. (2021), the Poisson KL estimator outperforms PRE and PLE. This study modified the KL estimator to propose a new estimator called the Poisson Modified KL estimator (PMKL). The new estimator falls in the same class with the ridge, Liu and KL estimators since they possessed a single shrinkage parameter. We investigated the performance of the estimators with a simulation study and a real-life application. From the results, we observed that the new estimator consistently performed well in the presence of multicollinearity with the lowest MSE. Finally, the new estimator is more suitable to combat multicollinearity in the PRM.

Data availability
All data underlying the results are available as part of the article and no additional source data are required.

---

**Table 6. Simulation result for mean square error (MSE) when P = 8 and intercept = -1.**

<table>
<thead>
<tr>
<th>β₀</th>
<th>n</th>
<th>ρ</th>
<th>MLE</th>
<th>PRE</th>
<th>PLE</th>
<th>PKL</th>
<th>PMKL1</th>
<th>PMKL2</th>
<th>PMKL3</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>50</td>
<td>0.8</td>
<td>0.8248</td>
<td>0.259</td>
<td>0.6469</td>
<td>0.4512</td>
<td>0.2159</td>
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</tr>
<tr>
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<td></td>
<td>0.9</td>
<td>1.1355</td>
<td>0.4945</td>
<td>0.8253</td>
<td>0.5699</td>
<td>0.4693</td>
<td>0.5526</td>
<td>0.9927</td>
</tr>
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<td></td>
<td></td>
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<td>1.7701</td>
<td>0.6264</td>
<td>1.1745</td>
<td>0.6921</td>
<td>0.5848</td>
<td>0.5614</td>
<td>1.4256</td>
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<td>1.6865</td>
<td>4.2093</td>
<td>1.0061</td>
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<td></td>
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<td>12.8251</td>
<td>38.0726</td>
<td>37.7305</td>
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<td></td>
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<td>0.3392</td>
<td>0.3019</td>
<td>0.2354</td>
<td>0.2115</td>
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<td></td>
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<td>0.9</td>
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</tr>
<tr>
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<td></td>
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<td>0.0810</td>
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<td>0.0693</td>
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<td>0.0813</td>
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<td>2.0665</td>
<td>0.2433</td>
<td>0.1621</td>
<td>1.7855</td>
</tr>
</tbody>
</table>

MLE = Maximum Likelihood Estimator; PRE = Poisson Ridge Estimator; PLE = Poisson Liu Estimator; PKL = Poisson Kibria-Lukman Estimator; PMKL1 = Poisson Modified Kibria Lukman Estimator 1; PMKL2 = Poisson Modified Kibria Lukman Estimator 2; PMKL3 = Poisson Modified Kibria Lukman Estimator 3.

**Table 7. Regression coefficients and MSE.**

<table>
<thead>
<tr>
<th>coef.</th>
<th>MLE</th>
<th>PRE</th>
<th>PLE</th>
<th>PKL</th>
<th>PMKL1 (k1)</th>
<th>PMKL2 (k2)</th>
<th>PMKL3 (k3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₀</td>
<td>-0.406</td>
<td>-0.167</td>
<td>-0.255</td>
<td>-0.107</td>
<td>-0.019</td>
<td>-0.002</td>
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<tr>
<td>a₁</td>
<td>0.569</td>
<td>0.380</td>
<td>0.479</td>
<td>0.391</td>
<td>0.120</td>
<td>0.179</td>
<td>0.322</td>
</tr>
<tr>
<td>a₂</td>
<td>0.165</td>
<td>0.171</td>
<td>0.167</td>
<td>0.168</td>
<td>0.183</td>
<td>0.179</td>
<td>0.172</td>
</tr>
<tr>
<td>a₃</td>
<td>-0.014</td>
<td>-0.015</td>
<td>-0.015</td>
<td>-0.016</td>
<td>-0.017</td>
<td>-0.017</td>
<td>-0.016</td>
</tr>
<tr>
<td>MSE</td>
<td>1.029</td>
<td>0.273</td>
<td>0.432</td>
<td>0.225</td>
<td>0.083</td>
<td>0.095</td>
<td>0.092</td>
</tr>
</tbody>
</table>

MLE = Maximum Likelihood Estimator; PRE = Poisson Ridge Estimator; PLE = Poisson Liu Estimator; PKL = Poisson Kibria-Lukman Estimator; PMKL1 = Poisson Modified Kibria Lukman Estimator 1; PMKL2 = Poisson Modified Kibria Lukman Estimator 2; PMKL3 = Poisson Modified Kibria Lukman Estimator 3.

**Conclusion**
The parameters in the PRM are commonly estimated using the Maximum Likelihood Estimator. However, literature had shown that the estimator suffers a setback when the explanatory variables are correlated. This problem led to the implementation of alternative estimators with single shrinkage parameters such as the Poisson Ridge Regression Estimator (PRE), Poisson Liu Estimator (PLE) and the Poisson KL Estimator (PKL). The KL estimator was generally preferred to the ridge regression and Liu estimator in the linear regression model. According to Lukman et al. (2021), the Poisson KL estimator outperforms PRE and PLE. This study modified the KL estimator to propose a new estimator called the Poisson Modified KL estimator (PMKL). The new estimator falls in the same class with the ridge, Liu and KL estimators since they possessed a single shrinkage parameter. We investigated the performance of the estimators with a simulation study and a real-life application. From the results, we observed that the new estimator consistently performed well in the presence of multicollinearity with the lowest MSE. Finally, the new estimator is more suitable to combat multicollinearity in the PRM.

**Data availability**
All data underlying the results are available as part of the article and no additional source data are required.
References


Open Peer Review

Current Peer Review Status:  ?  ?  ?

Version 1

Reviewer Report 09 August 2021

https://doi.org/10.5256/f1000research.57427.r89263

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Nimet Özbay
Department of Statistics, Faculty of Science and Letters, Çukurova University, Adana, Turkey

This article focuses on proposing a modified KL estimator to mitigate the Poisson Regression Model with multicollinearity. Some theoretical properties of the new estimator are examined. A numerical example is conducted to show the performance of the new estimator. I think the authors should give the definition of the modified KL estimator in more detail and explain its statistical necessity. The organization of the paper, grammatical mistakes, and punctuation errors should also be controlled. This article may be indexed after the major comments below are applied.

Comments:

The main document of the article does not contain line numbers, thus it has been quite difficult to pinpoint the location of the comments.

1. There are lots of language and punctuation errors throughout the whole article, so the authors should recheck the writing of the manuscript. For example:

   o In the Abstract, “the simulation result showed...” should be changed to “the simulation result showed that...”.

   o On page 3, a dot is required before equation (2.1).

   o On page 3, a comma is required before equation (2.2).

   o On page 5, line 1, “estimator” should be “estimators”.

   o Section number is required for the sections, etc.

   o The basic punctuation marks are missing throughout the paper.

2. On page 3, the abbreviation “PRE” is repeated.
3. In the whole article, there are some inappropriate uses of the abbreviations. The authors should rearrange the use of abbreviations. In some places, previously made abbreviations are repeated.

4. The use of “hat” is missing while presenting some estimators.

5. In the Introduction section, the manuscript *Defining a two-parameter estimator: a mathematical programming evidence* by Üstündağ Şiray *et al.* (2021)¹ may be mentioned since this is a more recent article in which a new biased estimator is proposed to mitigate multicollinearity.

6. On page 4, the authors should explain what lambdas are.

7. On page 6, before equation (2.8), “means square error” should be “MSE”.

8. There is no explanation for equation (2.11).

9. I think, “equation (2.10)” should be “equation (2.9)” before equation (2.12) on page 4.

10. On page 5, the authors should explain what lambdas are. Do the authors use “*V*” to show variance? If so, some explanations should be added about it.

11. On page 5, there is the incorrect use of “MSEM”. This abbreviation does not exist, although it is used while representing the lemmas and theorems.

12. I think the authors employ the canonical form in the proof of the theorems. Unfortunately, I did not find some information about the canonical model.

13. The selection of the biasing parameter section is insufficient. A detailed derivation and more information should be given.

14. In the simulation section, on page 7, why does the mean square error increase as the sample size increases?

15. It would be better if no abbreviations were used in the title.

References

Publisher Full Text

Is the work clearly and accurately presented and does it cite the current literature?
Partly

Is the study design appropriate and is the work technically sound?
Yes

Are sufficient details of methods and analysis provided to allow replication by others?
Partly

If applicable, is the statistical analysis and its interpretation appropriate?
Yes

Are all the source data underlying the results available to ensure full reproducibility?
Partly

Are the conclusions drawn adequately supported by the results?
Partly

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Monte Carlo simulation, Linear regression model, Econometric models, Applied statistics, Biased estimation

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

Reviewer Report 03 August 2021

https://doi.org/10.5256/f1000research.57427.r89258

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Muhammad Amin
Department of Statistics, University of Sargodha, Sargodha, Pakistan

In this paper, the authors introduced a new estimator by modified KL estimator for the Poisson regression model to overcome the effect of multicollinearity. The paper is original and deals with a topic of interest. This paper could be accepted for indexing after incorporating the following points.

- Write one paragraph on count data models and their importance at the start of the Introduction and include some citations that demonstrate the importance of count data models, for example: Amin et al., 2020; Amin et al., 2021; Sami et al., 2021; Amin et al., 2021; Majid et al., 2021; Rashad et al., 2019; Algamal et al., 2015; Algamal et al., 2021; Alanaz et al., 2018.

- Write the reason for your proposed estimator over other estimators in the last paragraph of
the Introduction section.

- Change independent variables to explanatory variables in the whole study.
- Write the first paragraph clearly and correct equation 2.1 of the Methods section by following Amin et al., 2020.
- Change “mean square error” to “mean squared error” in the whole manuscript.
- On page 3, write the reason for adapting the Poisson ridge estimator.
- On page 4, line 1, write the range of ridge parameter k.
- Write the limitations of the ridge estimator after equation (2.5).
- Write the range of Liu parameter d after equation (2.6).
- Write different notations of the ridge parameter k, for ridge, KL, and MKL estimators and also mention the ranges of these biasing parameters.
- Write the expressions for MSEs of ridge, Liu, and KL estimators.
- In Lemma 2.2, define b1, b2,
- The statement of Theorem 2.2 is wrong, I suggest the authors correct this.
- Define e in equation (2.23).
- In equation (2.24), change $\lambda_i$ to $\lambda_j$.
- Correct expressions above equation (3.1).
- The interpretations of simulation results need more detailed discussion.
- In real application, report the estimated values of each biasing parameter with proper citation of equations. Moreover, cite equation to compute MSE of the consider estimators.
- There are some grammatical issues that should be corrected.

References
Experience, 2021. Publisher Full Text

**Is the work clearly and accurately presented and does it cite the current literature?**
Partly

**Is the study design appropriate and is the work technically sound?**
Yes

**Are sufficient details of methods and analysis provided to allow replication by others?**
Yes

**If applicable, is the statistical analysis and its interpretation appropriate?**
Partly

**Are all the source data underlying the results available to ensure full reproducibility?**
Partly

**Are the conclusions drawn adequately supported by the results?**
Yes

**Competing Interests:** No competing interests were disclosed.

**Reviewer Expertise:** Regression Analysis, Biased Estimation Methods

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

Reviewer Report 21 July 2021

https://doi.org/10.5256/f1000research.57427.r89260
Mohammad Arashi
Department of Statistics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad, Mashhad, Iran

The paper extends the Liu estimator in generalized linear modeling. Specifically, the authors propose a new biased estimator for the estimation of regression coefficients in the discrete Poisson regression.

The results are interesting and the topic is eye-catching. The theoretical results are well supported by extensive numerical analysis.

I suggest the authors make minor revisions to improve the presentation before indexing.
1. Check the notation entirely to be consistent. For instance, in equation (2.10), “hat” must be added for the estimator. It happens also for (2.12).

2. Use another notation for diagonal matrices in equations. For example, you may use “L” and then define the elements.

3. Explain equation (2.21) more. Is the minimization over i?

4. In the simulation study, for the design generation, I suggest using another notation for “z_{ip}” since it is not the last element of the series of generated independent normals.

5. Provide a reference for the accessibility of the real data use in the real-life application.

Is the work clearly and accurately presented and does it cite the current literature? Yes

Is the study design appropriate and is the work technically sound? Yes

Are sufficient details of methods and analysis provided to allow replication by others? Yes

If applicable, is the statistical analysis and its interpretation appropriate? Yes

Are all the source data underlying the results available to ensure full reproducibility? Partly

Are the conclusions drawn adequately supported by the results? Yes
**Competing Interests:** No competing interests were disclosed.

**Reviewer Expertise:** High-dimensional modeling; shrinkage estimation

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

Author Response 27 Jul 2021

**BENEDICTA Aladeitan**, Landmark University, Omu-Aran, Nigeria

Thanks for your observations and corrections. All will be duly implemented.

**Competing Interests:** No competing interests were disclosed.

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